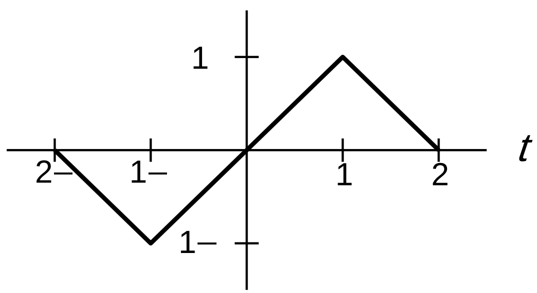
- Consider a transformation that maps the input signal x to the output signal y as given by

$$y(t) = x(at - b),$$

  where a and b are real numbers and $a \neq 0$.
- The above transformation can be shown to be the combination of a time-scaling operation and time-shifting operation.
- Since time scaling and time shifting *do not commute*, we must be particularly careful about the order in which these transformations are applied.
- The above transformation has two distinct but equivalent interpretations:
  1. first, time shifting x by b, and then time scaling the result by a; first,
  2. time scaling x by a, and then time shifting the result by b/a.
- Note that the time shift is not by the same amount in both cases.

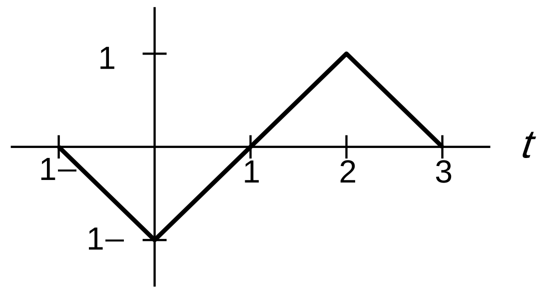
Given $x(t)$ as shown below, find $x(2t-1)$.

$x(t)$

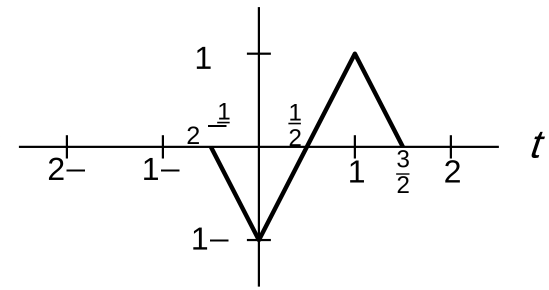


time shift by 1 and then time scale by 2

$p(t) = x(t-1)$

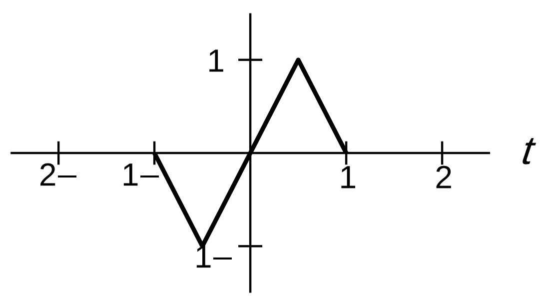


$p(2t) = x(2t-1)$

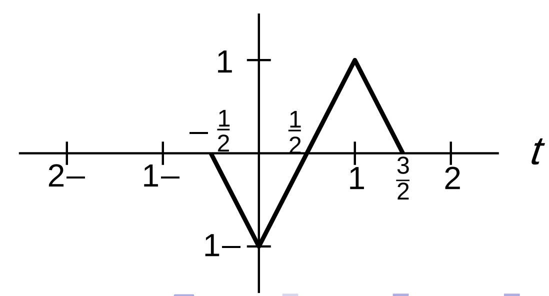


time scale by 2 and then time shift by $\frac{1}{2}$

$q(t) = x(2t)$



$q(t-1/2) = x(2(t-1/2))$
$= x(2t-1)$



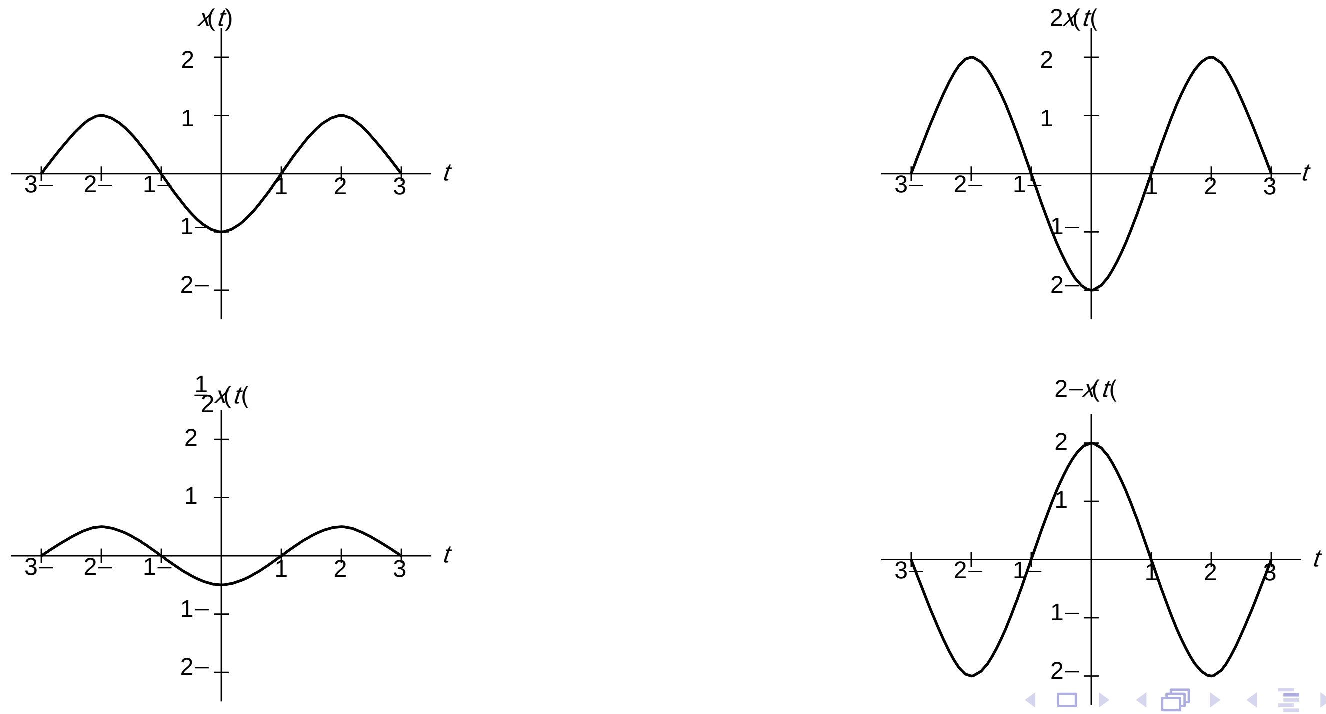
- A transformation of the independent variable can be viewed in terms of
  1. the effect that the transformation has on the *signal*; or
  2. the effect that the transformation has on the *horizontal axis*.
- This distinction is important because such a transformation has *opposite* effects on the signal and horizontal axis.
- For example, the (time-shifting) transformation that replaces t by $t-b$)where b is a real number) in $x(t)$ can be viewed as a transformation that
  1. shifts the signal x *right* by b units; or
  2. shifts the horizontal axis *left* by b units.
- In our treatment of independent-variable transformations, we are only interested in the effect that a transformation has on the *signal*.
- If one is not careful to consider that we are interested in the signal perspective (as opposed to the axis perspective), many aspects of independent-variable transformations will not make sense.

- Amplitude scaling maps the input signal x to the output signal y as given by

$$y(t) = ax(t),$$

where a is a real number.

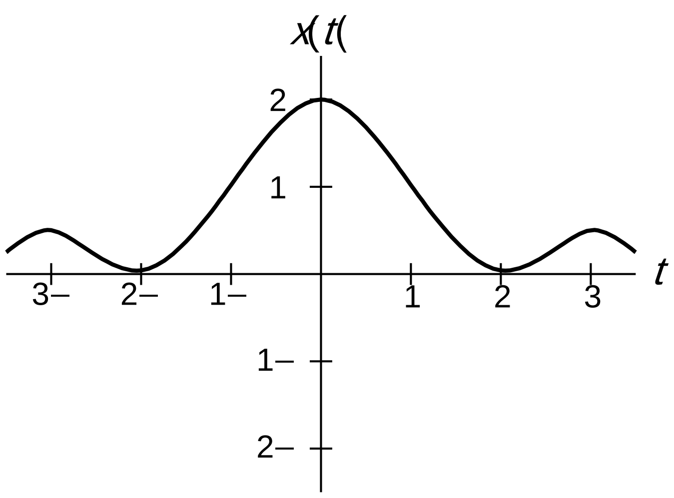
- Geometrically, the output signal y is *expanded/compressed* in amplitude and/or *reflected* about the horizontal axis.

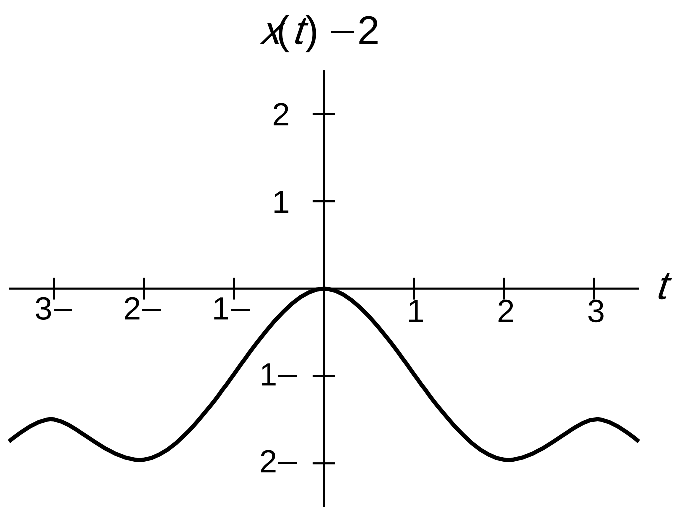


- Amplitude shifting maps the input signal x to the output signal y as given by

$$y(t) = x(t) + b,$$

  where b is a real number.
- Geometrically, amplitude shifting adds a *vertical displacement* to x.





- We can also combine amplitude scaling and amplitude shifting transformations.
- Consider a transformation that maps the input signal x to the output signal y, as given by

$$y(t) = ax(t) + b,$$

  where a and b are real numbers.
- Equivalently, the above transformation can be expressed as

$$y(t) = a\left[x(t) + \tfrac{b}{a}\right].$$

- The above transformation is equivalent to:
  1. first amplitude scaling x by a, and then amplitude shifting the resulting signal by b, or
  2. first amplitude shifting x by b/a, and then amplitude scaling the resulting signal by a.

Section 2.2

Properties of Signals

- Sums involving even and odd functions have the following properties:
  - The sum of two even functions is even.
  - The sum of two odd functions is odd.
  - The sum of an even function and odd function is neither even nor odd, provided that neither of the functions is identically zero.
- That is, the *sum* of functions with the *same type of symmetry* also has the *same type of symmetry*.
- Products involving even and odd functions have the following properties:
  - The product of two even functions is even.
  - The product of two odd functions is even.
  - The product of an even function and an odd function is odd.
- That is, the *product* of functions with the *same type of symmetry* is *even*, while the *product* of functions with *opposite types of symmetry* is *odd*.

- Every function x has a *unique* representation of the form

$$x(t) = x_e(t) + x_o(t),$$

  where the functions x_e and x_o are *even* and *odd*, respectively.
- In particular, the functions x_e and x_o are given by

$$x_e(t) = \tfrac{1}{2}\left[x(t) + x(-t)\right] \quad \text{and} \quad x_o(t) = \tfrac{1}{2}\left[x(t) - x(-t)\right].$$

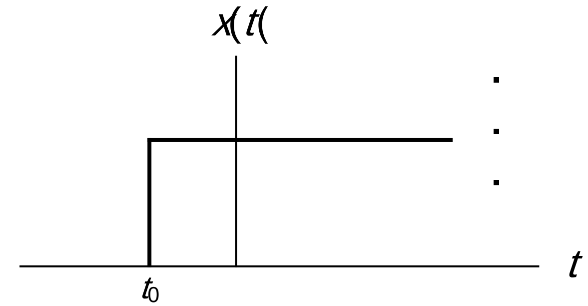
- The functions x_e and x_o are called the even part and odd part of x, respectively.
- For convenience, the even and odd parts of x are often denoted as $\mathrm{Even}\{x\}$ and $\mathrm{Odd}\{x\}$, respectively.

- **Sum of periodic functions.** Let x_1 and x_2 be periodic functions with fundamental periods T_1 and T_2, respectively. Then, the sum $y = x_1 + x_2$ is a periodic function if and only if the ratio *T_1/T_2 is a rational number* (i.e., the quotient of two integers). Suppose that $T_1/T_2 = q/r$ where q and r are integers and *coprime* (i.e., have no common factors), then the fundamental period of y is rT_1 (or equivalently, qT_2, since $rT_1 = qT_2$). (Note that rT_1 is simply the least common multiple of T_1 and T_2.)
- Although the above theorem only directly addresses the case of the sum of two functions, the case of N functions (where $N > 2$) can be handled by applying the theorem repeatedly $N-1$ times.

- A signal x is said to be right sided if, for some (finite) real constant t_0, the following condition holds:

$$x(t) = 0 \quad \text{for all } t < t_0$$

(i.e., x is *only potentially nonzero to the right of* t_0).
- An example of a right-sided signal is shown below.



- A signal x is said to be causal if

$$x(t) = 0 \quad \text{for all } t < 0.$$

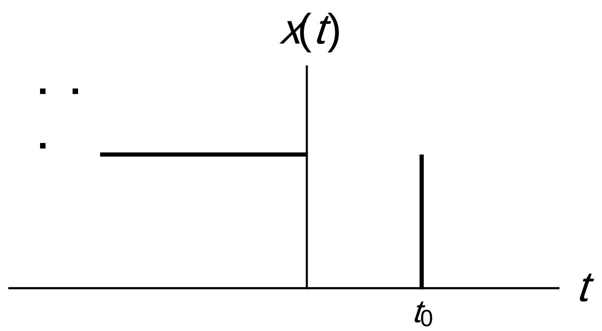
- A causal signal is a *special case* of a right-sided signal.
- A causal signal is not to be confused with a causal system. In these two contexts, the word "causal" has very different meanings.

- A signal x is said to be left sided if, for some (finite) real constant t_0, the following condition holds:

$$x(t) = 0 \quad \text{for all } t > t_0$$

(i.e., x is *only potentially nonzero to the left of* t_0).

- An example of a left-sided signal is shown below.

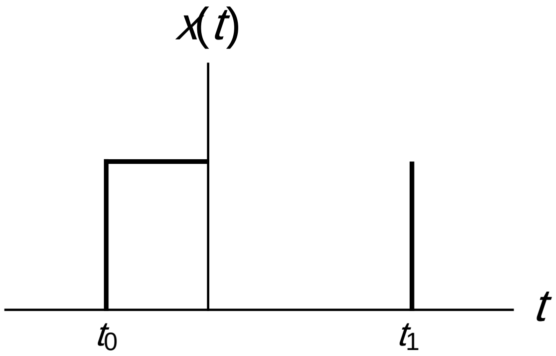


- Similarly, a signal x is said to be anticausal if

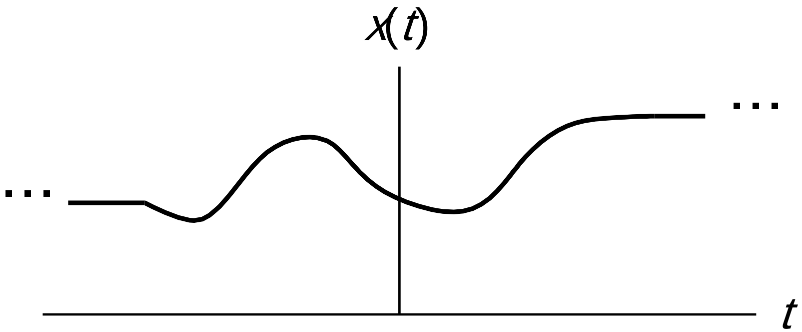
$$x(t) = 0 \quad \text{for all } t > 0.$$

- An anticausal signal is a *special case* of a left-sided signal.
- An anticausal signal is not to be confused with an anticausal system. In these two contexts, the word "anticausal" has very different meanings.

- A signal that is both left sided and right sided is said to be finite duration (or time limited).
- An example of a finite duration signal is shown below.



- A signal that is neither left sided nor right sided is said to be two sided.
- An example of a two-sided signal is shown below.



- A signal x is said to be bounded if there exists some (*finite*) positive real constant A such that

$$|x(t)| \leq A \quad \text{for all } t$$

-)i.e., $x(t)$ is *finite* for all t.(
- Examples of bounded signals include the sine and cosine functions.
- Examples of unbounded signals include the $\tan$ function and any
- nonconstant polynomial function.

- The energy E contained in the signal x is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

- A signal with finite energy is said to be an energy signal.
- The average power P contained in the signal x is given by

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt.$$

- A signal with (nonzero) finite average power is said to be a power signal.